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An Inclusive Cross Section for the Nucleus - Nucleus Interaction at RHIC Energies

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Abstract

We discuss the saturation of the parton density in heavy ion collisions at RHIC energies using a Pomeron approach. Our predictions for the particle density in ion-ion collisions at RHIC energies can be utilized as the background for the observation of possible quark-gluon plasma production.

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For heavy ion collisions at high energy, saturation of the parton density is predicted both in high density QCD [1, 2, 3] and in a Pomeron approach [5]. The goal of this letter is to estimate the value of the inclusive cross section for the case of nucleus-nucleus interactions in a Pomeron approach, as this will be the background for any interesting signal of new physics such as the saturation of the gluon density [1, 2, 3], or the creation of a quark-gluon plasma at RHIC [4]. The physical picture, behind our Pomeron approach, is the old fashioned parton model [6], which was and can still be used as a guide in strong interaction physics at large distances ("soft" physics). In spite of the simple physics described by Pomeron exchanges, the Pomeron approach is technically rather complicated, and this problem has not yet been solved for hadron-hadron interactions. However, for inclusive production in ion-ion collisions there are two significant simplifications which enable one to develop a self consistent theoretical description in our Pomeron approach:

1. For a heavy nucleus the vertex of the Pomeron-nucleus interaction $g_{PA}(b_t) = A^{\frac{1}{3}}\, g_{PN}(b_t = 0)\, S_A(b_t)$ where $g_{PN}(b_t = 0)$, is the vertex of the Pomeron - nucleon interaction at $b_t = 0$, and $S_A(b_t)$ is the nucleus profile function normalized so that $\int d^2b_t\, S_A(b_t) = 1$. Due to $A^{\frac{1}{3}}$ enhancement of Pomeron-nucleus interaction we can develop a theoretical approach [7] considering $G_{3P}\, g_{PA}(b_t = 0) \approx 1$ while $G^2_{3P} \ll 1$. Here, G_{3P} is the triple Pomeron vertex (see Fig. 1). In this approach one can neglect the loop Pomeron diagrams in comparison with the "tree" diagrams of Fig.1.

2. Using the AGK-cutting rules [8], all Pomeron diagrams in which more then two Pomerons cross the rapidity level y_c cancel in the inclusive cross section. Therefore, only Mueller diagrams, shown in Fig.1, survive.

Using the approach, developed in Refs.[7, 5, 9], we obtain the following closed expression for the diagrams shown in Fig.1 (see Fig.1 for notation)

$$\frac{d\sigma(A_1 + A_2)}{dy_c} = \int d^2b_t\, d^2b'_t\, a^P_P\, R_{A_1}\left(Y - y_c, b_t\right) \cdot R_{A_2}\left(y_c, b'_t\right) = \tag{1}$$

$$= a^P_P \int d^2b_t\, d^2b'_t\, \frac{g_{PA_1}(b_t)\, g_{PA_2}(b'_t)\, e^{\Delta Y}}{\left(g_{PA_1}(b_t)\,\gamma\,(e^{\Delta\,(Y - y_c)} - 1) + 1\right)\ \left(g_{PA_2}(b'_t)\,\gamma\,(e^{\Delta\, y_c} - 1) + 1\right)}$$

here $\gamma = G_{3P}/\Delta$ and G_{3P} is the vertex of the triple Pomeron interaction, $\Delta = \alpha_P(0) - 1$ where $\alpha_P(0)$ is the Pomeron's intercept, a^P_P is the particle emission vertex and $Y = \ln\left(s/1\,GeV^2\right)$ is the total rapidity interval for $A_1 - A_2$ collision. $R_A(Y, b_t)$ is the sum of "tree" diagrams and is given in [7, 5].

Eq.(1) predicts saturation of the density of produced particles which can be defined as

$$\rho\,(y_c) = \frac{d\,N(y_c)}{d\,y_c} = \frac{\frac{d\sigma(A_1+A_2)}{dy_c}}{\sigma_{tot}(A_1 + A_2)}, \tag{2}$$

where $N(y_c)$ denotes the multiplicity of particles with rapidity y_c.

Eq.(1) leads to a constant density $\rho(y_c)$ at high energies. To show this, we note that using a simplified Wood-Saxon type b_t distribution for the density of a nucleus, namely,

$$S_A\,(b_t) = \frac{1}{\pi\, R^2_A}\,\Theta\,(R_A - b_t)\ , \tag{3}$$

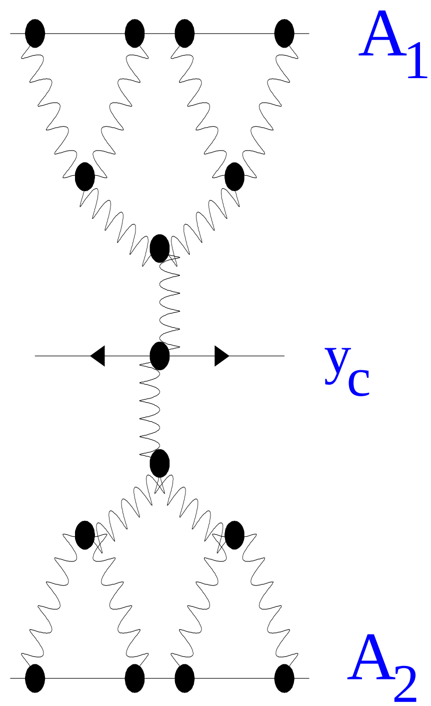

Figure 1: *The Mueller diagram for an inclusive A-A interaction process.*

the total cross section (for $A_1 > A_2$) is

$$\sigma_{tot} = 2\,\pi\,R^2_{A_1}. \tag{4}$$

Integrating over b_t and b'_t in Eq.(1) we have

$$\rho(y_c) \ \longrightarrow |_{Y \gg y_c \gg 1} \quad a^P_P \, \frac{R^2_{A_2}}{2\pi\ \gamma^2}\ , \tag{5}$$

where at high energy (see Fig.1)

$$a^P_P = \frac{\frac{d\sigma(N+N)}{dy_c}}{\sigma_{tot}(N+N)}$$

Eq.(5) suggests that the particle density is independent of the energy and of the atomic number of the heaviest nucleus. It is instructive to compare this prediction with the density obtained in the Glauber-Gribov approach [10] where

$$\rho(y_c) \ \longrightarrow |_{y \gg 1}\ a^P_P\ A_2 A_1^{\frac{1}{3}}\ e^{\Delta\ Y}\ . \tag{6}$$

Eq.(6) predicts a different behaviour for the particle density than does Eq.(5), with a clear dependence on energy and on A_1. Eq. (1) also contains a very simple prediction for the particle density in the fragmentation regions. Assume that $y_c \to 0$ and $y_c \to Y$ correspond to the fragmentation regions for lightest and heaviest nucleus, respectively. From Eq. (1) it follows that

$$\rho(y_c)\ |_{y_c \to 0} \quad = \quad A_2\ \frac{g_{PN}\,\gamma}{2\pi}\ ; \tag{7}$$

$$\rho(y_c)\ |_{y_c \to Y} \quad = \quad A_2^{\frac{2}{3}}\ A_1^{\frac{1}{3}}\ \frac{g_{PN}\,\gamma}{2\pi}\ . \tag{8}$$

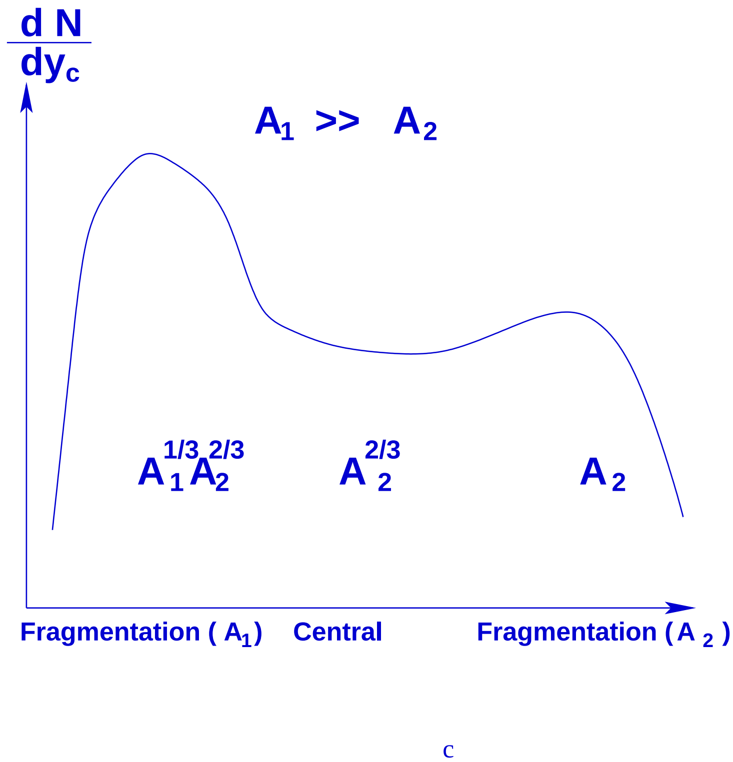

Figure 2: *Our prediction for particle production.*

Fig. 2 illustrates our prediction for particle density.

The physics that Eq. (5) is based on, is the same saturation of the parton density that has been discussed over the past decade in the framework of high density QCD [1, 2, 3]. Namely, due to the emission of partons, the parton density increases at high energies but, simultaneously, the probability of the interaction of two partons increases as well. Since, two interacting partons can annihilate into one, such interactions diminish the number of partons in the parton cascade. Finally, competition between the emission and the annihilation leads to an equilibrium state with a definite value for the parton density. We call this phenomenon, saturation of the parton density. The high density QCD approach leads to the following $A - A$ dependence of the process [1, 2, 3]:

$$< N_{A_1\ A_2} >= Q_s^2\,(s, A_1) \cdot A_2^{\frac{2}{3}} \cdot < N_{N\ N} >,$$

here $A_2 < A_1$, $Q_s^2\,(s, A_1)$ is the saturation scale, which increases with A, and the energy squared s, $< N >$ is the density of the produced hadrons in the $A_1 - A_2$ or the $N - N$ interactions. $Q_s^2\,(s, A)$ grows with A (at least $Q_s^2 \propto A^{\frac{1}{3}}$ [11], but it could even be proportional to $A^{\frac{2}{3}}$ [12]), and therefore high density QCD predicts

$$< N_{A_1\ A_2} >\propto A_2^{\frac{2}{3}} \cdot \left(A_1^{\frac{1}{3}} \div A_1^{\frac{2}{3}} \right), \tag{9}$$

where $A_2 < A_1$. Eq. (5) can be rewritten as:

$$< N_{A_1\ A_2} >= A_2^{\frac{2}{3}} < N_{N\ N} >, \tag{10}$$

where $A_2 < A_1$.

Eq. (10) leads to different predictions than those of both the Glauber approach (see Eq. (6)) and the high density QCD approach (see Eq. (9)).

To calculate the particle density at RHIC energies we need to specify how to calculate the total cross section. It turns out that even in our approach for nucleus-nucleus scattering, the problem of determining the total cross section is rather complicated, but it has been solved in Ref. [5]. We use the following expression for σ_{tot}:

$$\sigma_{tot} \; = \; 2 \int d^2\, b_t \, \left(1 \; - \; \exp \left(- \frac{\Omega(s, b_t)}{2} \right) \right) \; . \tag{11}$$

$$\tag{12}$$

Here $\Omega(s, b_t)$ the opacity is given by

$$\Omega(s, b_t) \; = \; \int d^2 b'_t \;\; F \left(Y, b'_t, |\vec{b}_t - \vec{b'}_t| \right) \; , \tag{13}$$

and $F \left(Y, b'_t, |\vec{b}_t - \vec{b'}_t| \right)$ is the amplitude of the nucleus-nucleus interaction [5]. Since the RHIC energies are not very high ($Y = \ln(s/1GeV^2) \approx 10$ we can use a simple formula for F (see Ref. [5] for details):

$$F\left(Y, b_t, b'_t\right) \; = \; \frac{g_{PA_1}(b_t - b'_t) \cdot g_{PA_2}(b'_t) \cdot e^{\Delta Y}}{\left(g_{PA_1}(b_t - b'_t) + g_{PA_2}(b'_t)\right) \gamma \left(e^{\Delta Y} - 1\right) + 1} \; . \tag{14}$$

We assume the Wood-Saxon parametrization [13] for profile function $S_A(b_t)$.

$$S_A(b_t) \; = \; \frac{\rho}{1 \; + \; e^{\frac{r - R_A}{h}}} \; ; \tag{15}$$

where R_A is the nucleus radius $R_A = 1.4 A^{\frac{1}{3}} \, fm$ and h the surface thickness $h \approx 1 fm$. For a numerical estimate we used the parameters of Eq. (15) given in Ref. [14].

It is even easier to calculate the particle density for central A-A collisions which has been measured at RICH. The observable can be written as a ratio of the inclusive cross section to the cross section at fixed $b_t = 0$.

$$\rho_{central}\left(y_c\right) \; = \; \frac{\int \;\; d^2 b'_t \; a^P_P \; R_{A_1}\left(Y - y_c, b'_t\right) \; \cdot \; R_{A_2}\left(y_c, b'_t\right)}{2 \; \left(1 \; - \; \exp\left(-\frac{\Omega(s, b_t = 0)}{2}\right)\right)} \; \rightarrow \tag{16}$$

$$\rightarrow \int \;\; d^2 b'_t \; \frac{g_{PA_1}(b'_t) \cdot g_{PA_2}(b'_t) \cdot e^{\Delta Y}}{\left(g_{PA_1}(b'_t) \gamma \left(e^{\Delta (Y - y_c)} - 1\right) + 1\right) \; \left(g_{PA_2}(b'_t) \gamma \left(e^{\Delta \, y_c} - 1\right) + 1\right)} \tag{17}$$

since $\Omega(s, b_t = 0) \gg 1$ for A-A scattering.

We calculate the particle density, defined be Eq. (2), for Au-Mo, Au-Ne and Ne-Mo scattering at the RHIC energies, $W = 56\, GeV$ ($Y = \ln\left(s/1\, GeV^2\right) = 8$) and $W = 156\, GeV$ ($Y = \ln\left(s/1\, GeV^2\right) = 10$) . For proton-proton scattering we use the following parameters, which fit the experimental data quite well (see Ref. [5] for details):

$$\gamma = 1.19 \; GeV^{-1} \quad g_{PN} = 8.4\, GeV^{-1} \quad \Delta = 0.07 \quad a^P_P = 2.$$

Fig. 3 shows that we find a particle density which is $2 \div 3$ times smaller than in the Glauber approach. The contrast is even more impressive when compared with the prediction of the gluon saturation approaches or/and for quark-gluon plasma production (see Refs.[15] and [16] for example). The physics is very simple and is the same as for the gluon saturation approach: the interactions between partons diminish their number, but in contrast to gluon saturation the interaction only occurs when the partons have low transverse momenta.

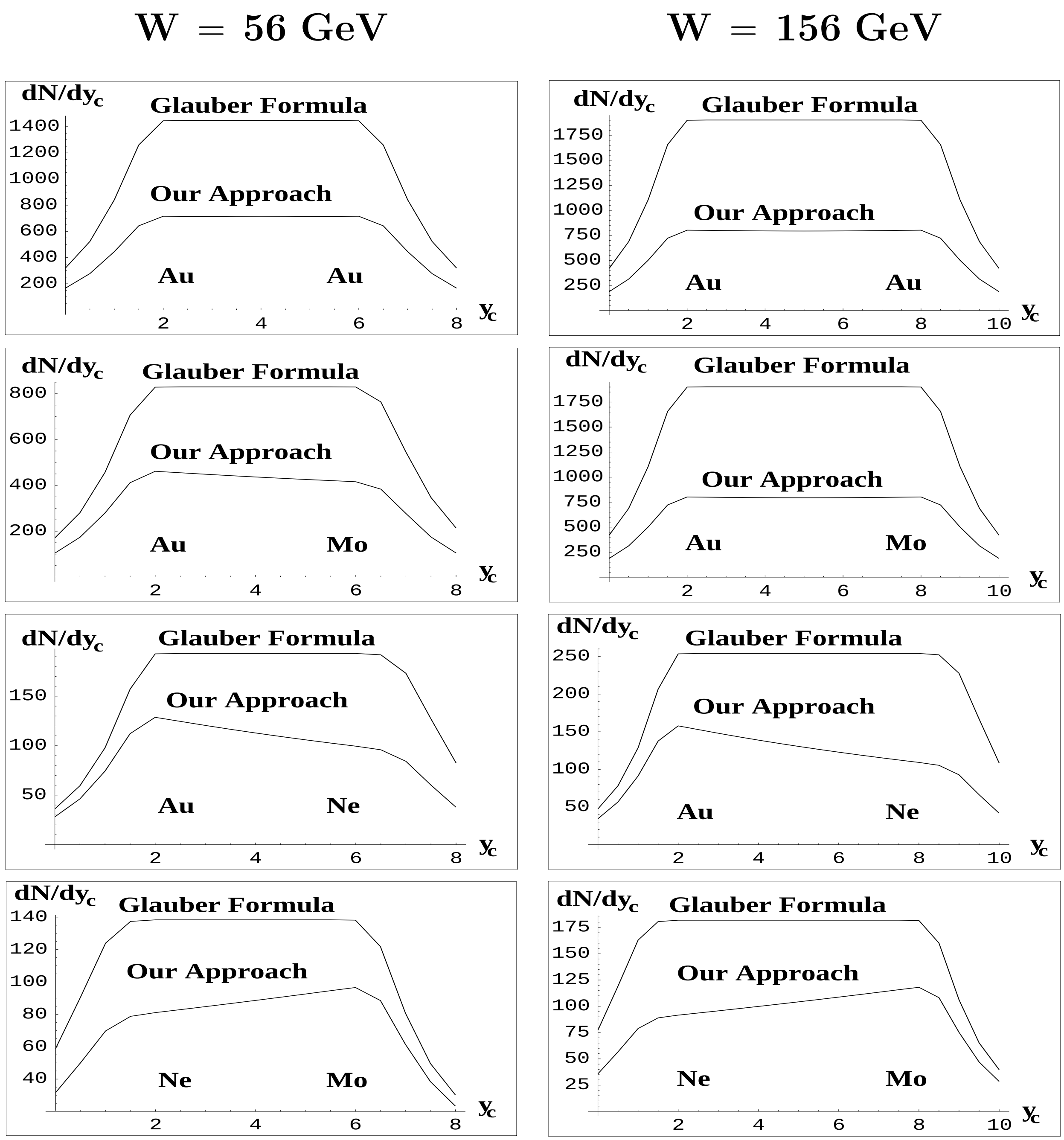

Figure 3: *The density of produced hadrons for Au-Au, Au-Mo, Au-Ne and Mo-Ne in central A-A interactions at RHIC energies.*

Comparison with the experimental data[17][1] is given in Fig.4. We plot three different theoretical curves in Fig.4: the upper one is the calculation of the hadron density for a central collision, calculated using Eq. (16) with $b_t = 0$; the middle curve is calculated , assuming that centrality of the nucleus interaction was fixed with an accuracy $b_t \leq 2\,fm$ [2]; and the lowest curve is the inclusive density of Eq. (2). One can see that the agreement both in value and in the energy dependence is rather good, but it depends strongly on the accuracy with which the centrality of the experimental interaction was determined. However, it is very difficult to distinguish our prediction for gold-gold collisions from those based on the saturation approach at high density QCD, when the saturation scale is low (approximately $1 \div 1.5\,GeV$) as was suggested in Ref. [16]. For this case both approaches describe the same physics at RHIC energies and we need more data on different nuclei to differentiate between these two approaches.

Our conclusions are simple. We show that the RHIC data on the rapidity multiplicity in the central A-A collision can be used to test the different models [1, 2, 3, 11, 15, 16]. We predict that at RHIC, densities will be rather small (400-500 particles per unit of rapidity for gold-gold interaction, see Fig.4) and these data support the saturation hypothesis, at least for sufficiently small transverse momenta for which our Pomeron approach is valid. Secondly, our calculations should be considered as a background for a signal of new physics such as saturation of the gluon density in high energy QCD or/and quark-gluon plasma production. Fig.3 shows that the measurement of A-dependence will be very decisive in distinguishing between the saturation due to the Pomeron interaction and the high density QCD approach.

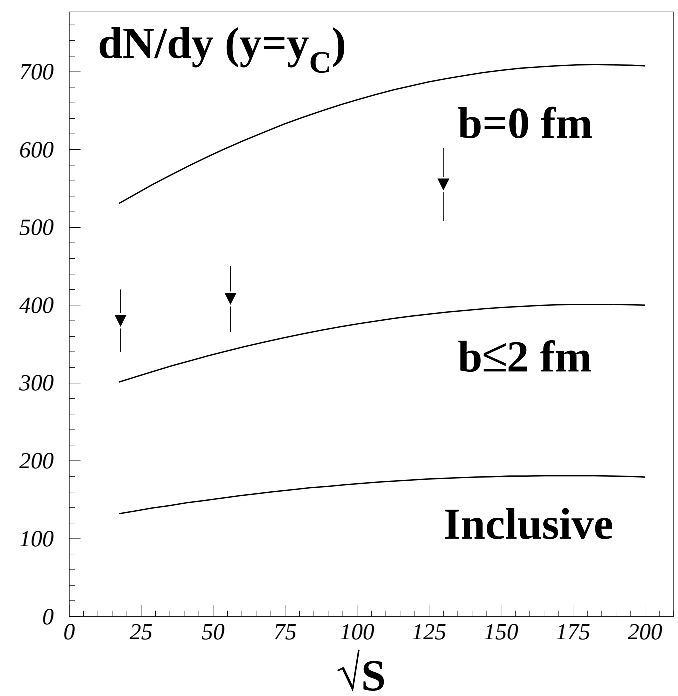

Figure 4: *Energy behaviour of the central hadron density.*

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[1] The experimental data at lowest energy ($\sqrt{s} = 24\ GeV$) were recalculated from lead - lead collision to gold-gold one , using our formulae.

[2] We integrated both the numerator and the denominator in Eq. (16) over $b_t \leq 2\,fm$.

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